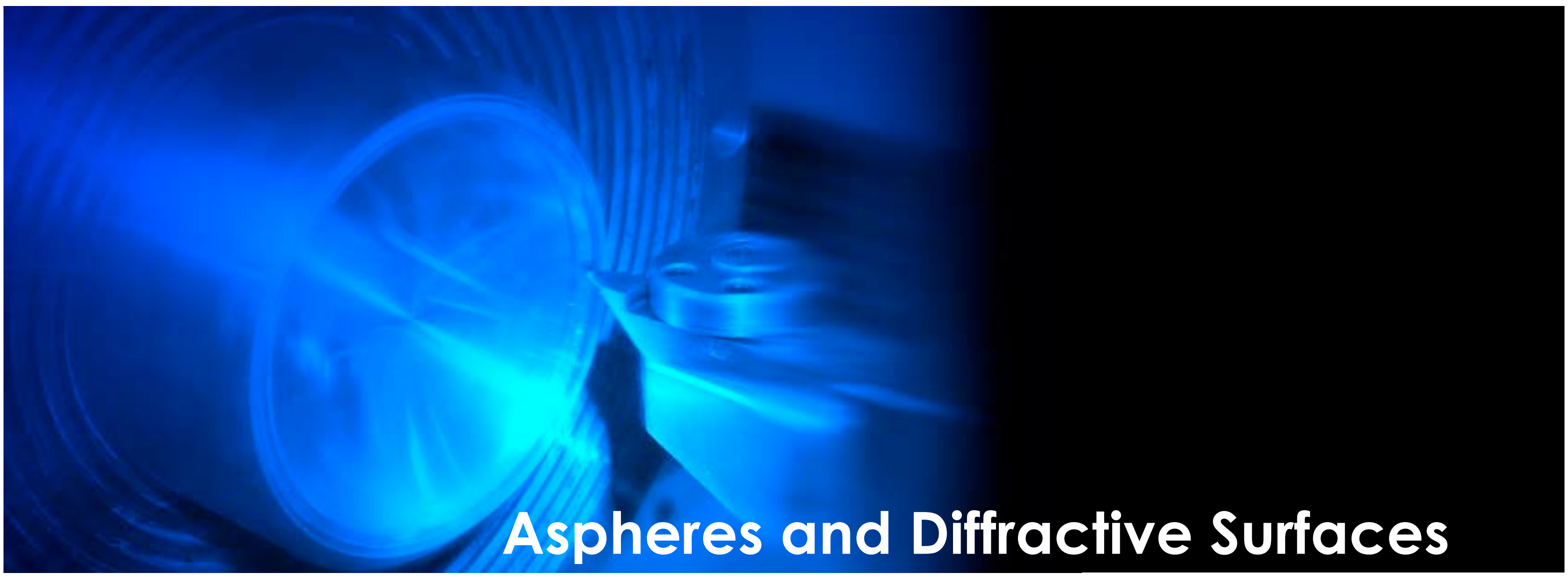

# Aspheres and Diffractive Surfaces



## The Confusion with Inches/Millimeters and the Phase Coefficient Relation Between Code V/Zemax

By Max J. Riedl

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### 1. Introduction

Sometimes, lens data are given in inches. This by itself is of course no problem. However, it is frequently overlooked that higher term coefficients for an aspheric surface change as well when the design data are changed from inches to metric (usually mm). Furthermore, if a surface is a diffractive one, there is much of a struggle to convert the phase coefficients from one program to another. In addition, there is occasionally a doubt about the direction of the phase structure, which can be devastating if the wrong one is taken.

These relations become clear on hand of a numerical example for which a 2 element LWIR Petzval objective has been chosen with a diffractive phase profile superimposed on the aspheric rear surface of the front element. The front surface of the second element is an asphere. The other surfaces of the lens elements are spheres. The objective is shown in Fig.1

### 2. The Objective

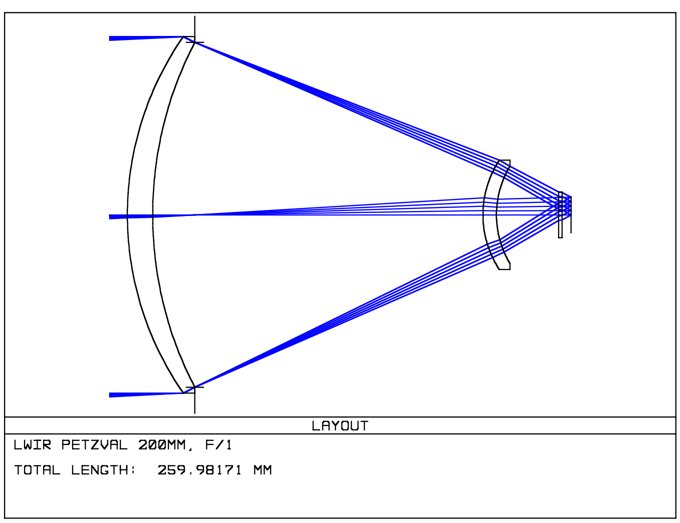


**Fig.1.** A typical LWIR objective with filter in front of the detector.

For our analysis here, we concern ourselves only with the dimensions and coefficients of the front element. These are presented in Table 1.

**2. Prescription of the front element in mm.**
**Phase coefficients as expressed in Code V program**

**Table 1**

| Item | Front | Rear |
|---|---|---|
| Surface type | Sphere | Asphere with diffractive phase profile |
| Radius R | 174.529 mm | 209.925 mm |
| Conic Constant K | 0 (Sphere) | 0 (Base surface is a sphere) |
| Aspheric coefficients | | |
| A4 or A | - | 0.180817 E-08 $mm^{-3}$ |
| A6 or B | - | 0.392629 E-13 $mm^{-5}$ |
| A8 or C | - | 0.356891 E-18 $mm^{-7}$ |
| A10 or D | - | 0.369300 E-22 $mm^{-9}$ |
| Phase coefficients | | |
| Power of 2 ($S_2$) | - | -0.581441 E-05 |
| Power of 4 ($S_4$) | - | -0.176653 E-10 |
| Design wavelength $\lambda_0$ | 10 μm | 10 μm |
| Index of refraction $N_{10}$ | 4.00312 | 4.00312 |

**3. Conversion of phase coefficients for Zemax program**

The general conversion equation for the phase coefficients from Code V to Zemax is

$$P_{2n} = \frac{2\pi}{\lambda_0} R_0^{2n} S_{2n}, \tag{1}$$

where $P_{2n}$ is the Zemax phase coefficient, $\lambda_0$ the design wavelength, $R_0$ the normalized radius, a feature used with Zemax. $S_2$ is the Code V phase coefficient. $n$ indicates the order. This will become clear in the following calculation for our lens.

**Important remark:** It is customary, but not necessary, to set $R$ = 100, when the lens dimensions are given in mm. The case for $R_0 \neq 100$ is discussed in section 10.

With that, $P_2 = \frac{2\pi}{0.01} 100^2 (-0.581441E-05) = -36.533015$.

(Notice, for this calculation, $n$ = 1)

For $P_4$ we use $n$ = 2, and obtain $P_4 = \frac{2\pi}{0.01} 100^4 \left(-0.176651E-10\right) = -1.109944$.

It is also well to remember to state the wavelength in mm if the lens dimensions are in mm. Therefore we inserted 0.01 mm for 10 μm.

**4. The zone radii**

The zone-or transition radii can be calculated from the phase equation, which is

$$i\lambda_0 = \left| S_2\rho^2 + S_4\rho^4 + \ldots S_j\rho^j \right|, \tag{2}$$

with $\rho$ being the zone radius, and $i$ = 1,2,3,….. the zone number.

Inserting the values from our example, we find for the first zone radius

$$1 \times 0.01 = \left| (-0.581441E-05)\rho^2 + (-0.176653E-10)\rho^4 \right|,$$

which leads to $\rho_1$ = 41.364 mm.

Solving for the rest of the zone radii yields:
$\rho_2$ = 58.348 mm,
$\rho_3$ = 71.282 mm,
$\rho_4$ = 82.106 mm, and
$\rho_5$ = 91.573 mm.

**5. Prescription of the front element in inches.**
**Phase coefficients as expressed in Code V program**

**Table 2**

| **Item** | **Front** | **Rear** |
|---|---|---|
| Surface type | Sphere | Asphere with diffractive phase profile |
| Radius R | 6.8712 inch | 8.2648 inch |
| Conic Constant K | 0 (Sphere) | 0 (Base surface is a sphere) |
| Aspheric coefficients | | |
| A4 or A | - | 0.296306 E-04 inch$^{-3}$ |
| A6 or B | - | 0.415098 E-06 inch$^{-5}$ |
| A8 or C | - | 0.243429 E-08 inch$^{-7}$ |
| A10 or D | - | 0.162511 E-09 inch $^{-9}$ |
| Phase coefficients | | |
| Power of 2 ($S_2$) | - | -0.147686 E-03 |
| Power of 4 ($S_4$) | - | -0.289483 E-06 |
| Design wavelength $\lambda_0$ | 10 $\mu$m | 10 $\mu$m |
| Index of refraction $N_{10}$ | 4.00312 | 4.00312 |

**6. Conversion of phase coefficients for Zemax program**

This time we have to insert the dimensions into Eq.(1) in inches. Therefore,

$$P_2 = \frac{2\pi}{0.0003937} 3.937008^2 (-0.147686E-0.3) = -36.533015\,.$$

Notice, the phase coefficient does not change for Zemax. It is the same as for the lens dimensioned in mm. Of course the wavelength as well as the normalized radius have to be inserted in inches.

**7. The zone radii**

For the first zone radius, Eq.(2) becomes

$$1 \times 0.0003937 = \left|(-0.147686E-03)\rho^2 + (-0.289483E-06)\rho^4\right|,$$

which yields $\rho_1$ = 1.62850 inch.

For the rest we find:
$\rho_2$ = 2.29717 inch,
$\rho_3$ = 2.80638 inch,
$\rho_4$ = 3.232521 inch, and
$\rho_5$ = 3.60525 inch.

Obviously, the conversion could have been done simply by multiplying the values from section 4 by 25.4. The purpose for the long way around was to demonstrate the relationship and correct use of the coefficients. To convert the aspheric coefficients $A_2$ through $A_{10}$ (or *A* through *D* as they are sometimes called) from the mm to the inch format, the following rule is applied

$$A_{n_{inch}} = A_{n_{mm}} \times 25.4^{(n-1)}. \tag{3}$$

For our example, $A_{4_{mm}} = 0.180817E-08\,\mathrm{mm}^{-3}$, therefore

$$A_{4_{inch}} = (0.180817E-08) \times 25.4^{(4-1)} = (0.180817E-0.8) \times 25.4^3 = 0.296306\,\mathrm{inch}^{-4}.$$

This confirms the number in Table 2.

**8. The step size at the zone transition**

The relation among the step size, the design wavelength, and the index of refraction of the lens material is given by

$$d = \frac{\lambda_0}{(N-1)}\ . \tag{4}$$

The material for the front element was Germanium with *N* = 4.00312. Therefore, for the design wavelength of $\lambda_0$=10 $\mu$m,

$$d = \frac{10}{4.00312-1} \cong 3.33\text{ìm}\ .$$



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**9. The direction of the phase profile and its effects**

The sagittal height of an asphere is expressed by the well known relation

$$z_{asph} = \frac{c\rho^2}{1+\sqrt{1-(K+1)c^2\rho^2}} + A_4\rho^4 + A_6\rho^6 + \ldots A_j\rho^j . \tag{5}$$

The sagittal height of a diffractive profile is stated by

$$z_{diffr} = \frac{1}{(N_0-1)}\left[S_2\rho^2 + S_4\rho^4 + \ldots S_j\rho^j\right] + \frac{\lambda_0}{(N_0-1)}\left[\left| Int\frac{1}{\lambda_0}\left(S_2\rho^2 + S_4\rho^4 + \ldots S_j\rho^j\right)\right|\right] \tag{6}$$

The symbols used in these expressions have been identified above, with the exception of $c$, which is $1/R$, the curvature of the surface.

The total sagittal height is simply the sum of both,

$$z_{total} = z_{asph} + z_{diffr} . \tag{7}$$

To demonstrate all this with numbers, we analyze the second surface of the front element of our Petzval objective. As a radial coordinate we pick $\rho = 75$ mm.

$$\begin{aligned} z_{asph} = & \frac{(1/209.925)\times 75^2}{1+\sqrt{1-(0+1)\times(1/209.925)^2\times 75^2}} \\ & + 0.180817\times 10^{-8}\times 75^4 + 0.392629\times 10^{-13}\times 75^6 \\ & + 0.356891\times 10^{-18}\times 75^8 + 0.3693\times 10^{-22}\times 75^{10} \end{aligned}$$

$z_{asph}$ = 26.795284 + 0.057212 + 0.006988 + 0.000357 +0.000208 = 26.86005 mm.

$$\begin{aligned} z_{diffr} = & \frac{1}{(4.00312-1)}\left[-0.581441\times 10^{-5}\times 75^2 - 0.176653\times 10^{-10}\times 75^4\right] \\ & + \frac{0.01}{(4.00312-1)}\left[\left| Int\frac{1}{0.01}\left(-0.581441\times 10^{-5}\times 75^2 - 0.176653\times 10^{-10}\times 75^4\right)\right|\right] \end{aligned}$$

$z_{diffr}$ = - 0.011077 + 0.009990 = - 0.00109 mm.

Therefore, $z_{total}$ = 26.86005 – 0.00109 = 26.85896 mm.

Here is where the mistake is sometimes made. The sign of $z_{diffr}$ is incorrectly applied. The effect for the correct and incorrect direction of the diffractive structure is shown in Fig. 2.

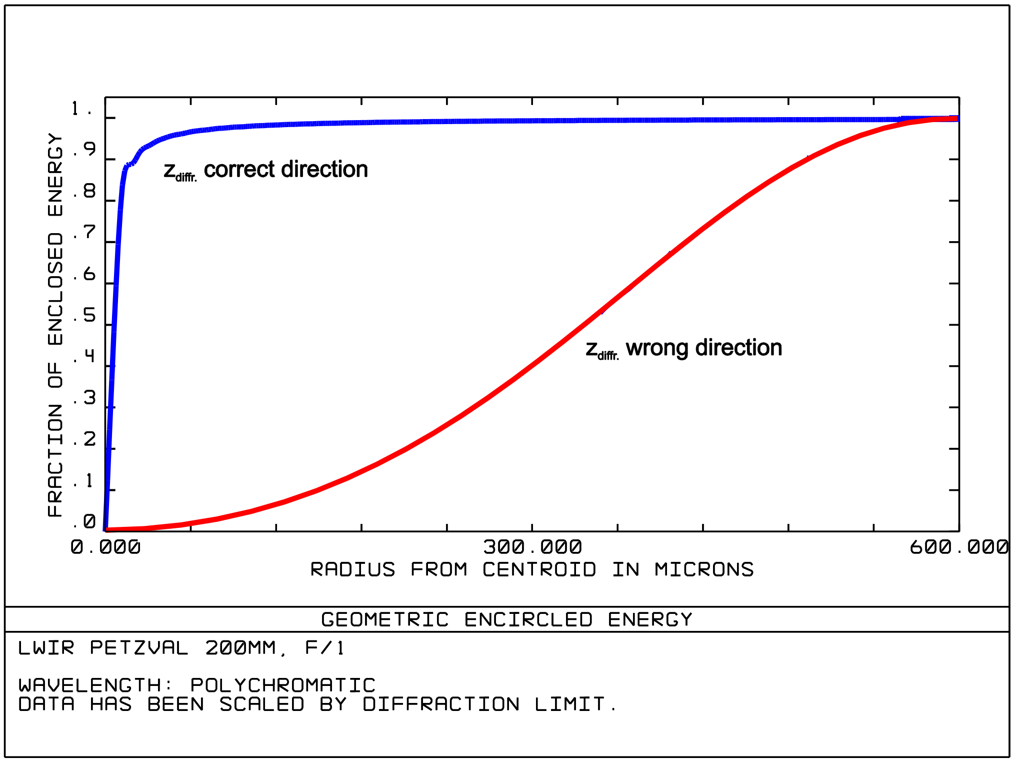


**Fig. 2.** Encircled energy for system with right and wrong direction of phase profile.

This completes the analysis of the 200 mm Petzval front element.

**10. Normalized radius for the Zemax program**

Eq.(1) $P_{2n} = \frac{2\pi}{\lambda_0} R_0^{2n} S_{2n}$ states clearly that the normalized radius $R_0$ needs to be known for the correct coefficient conversion from Code V to the Zemax or vice versa.

To incorporate the relationship in a general way into the standard phase expression, Eq.(2) changes to

$$i = \frac{1}{2\pi R_0^2} \left\| \left[ P_2 \rho^2 + \frac{P_4}{R_0^2} \rho^4 + \ldots\ldots \frac{P_n}{R_0^{(n-2)}} \rho^n \right] \right\| \tag{5}$$

Notice, the wavelength does not appear in this expression.

Let us demonstrate with an example. For a particular design the Zemax coefficient $P_2$ was given as –141.56377. The aperture radius was chosen as the normalized radius. It was $R$ = 1.825 inches. The design wavelength was 10 μm (0.0003937 inch).

Using Eq.(1), and solving for the Code V coefficient, yields

$$S_2 = \frac{0.0003937}{2\pi 1.825^2}(-141.455983) = -0.0026612 .$$

To calculate the first zone radius, we use Eq.(5). Since there is only a quadratic term we can write

$$\rho_i = \sqrt{\frac{2\pi R_0 i}{|P_2|}} = \sqrt{\frac{2\pi\ 1.825^2 i}{141.455983}} = 0.3846291\sqrt{i}\ .$$

With this, all the zone radii can be quickly calculated.

**Table 3**

| **i** | **ρ**(inches) | **i** | **ρ**(inches) |
|---|---|---|---|
| 1 | 0.3846 | 12 | 1.3324 |
| 2 | 0.5439 | 13 | 1.3868 |
| 3 | 0.6662 | 14 | 1.4392 |
| 4 | 0.7693 | 15 | 1.4897 |
| 5 | 0.8601 | 16 | 1.5385 |
| 6 | 0.9421 | 17 | 1.5859 |
| 7 | 1.0176 | 18 | 1.6318 |
| 8 | 1.0879 | 19 | 1.6766 |
| 9 | 1.1539 | 20 | 1.7201 |
| 10 | 1.2163 | 21 | 1.7626 |
| 11 | 1.2757 | 22 | 1.8041 |

### 11. Conclusion

It has been demonstrated that much care has to be taken when coefficients are to be converted from one program to another. This includes also the change from metric lens units to inches. Furthermore, close attention has to be paid to the direction of the phase profile.