Optimized Tool Path Descriptions For Aspherical Surface Generation

Mark Craig Gerchman
Rank Taylor Hobson Inc.
Keene, New Hampshire USA 03431-0543

The modern ultra-precise machine tool is capable of producing aspherical surface geometries to demanding tolerances. Many factors (*e.g.* machine tool accuracies, environmental effects, and machining dynamics) influence the accuracy of the surfaces generated. One factor that directly influences surface geometry is the description of the tool path used by the machine tool's numerical control system. This paper will describe a new algorithm for the optimization of the tool path description for aspherical surface generation.

Various interpolation schemes are used by ultra-precise machine tool control systems to "smooth" tool path descriptions. These interpolation schemes include linear, circular, polynomial, spline and machine specific descriptions. Simple linear interpolation has remained the preferred choice for the ultra-precision community since the use of multiple points provides for greater user control over path definition and compensation for residual form errors.[1]

Current software, that use a linear interpolation scheme to describe tool paths, define a series of points that are equally spaced along the desired theoretical curve and offset to account for tool tip geometry. These points are used by the machine tool control system to position the machine such that the relative positions of the tool and work piece follow the described straight line segments between these points and approximate the desired curve (see figure 1). The number of points used to define the curve must be selected to maintain a maximum difference between the linear segment and the theoretical curve below some specified tolerance limit.

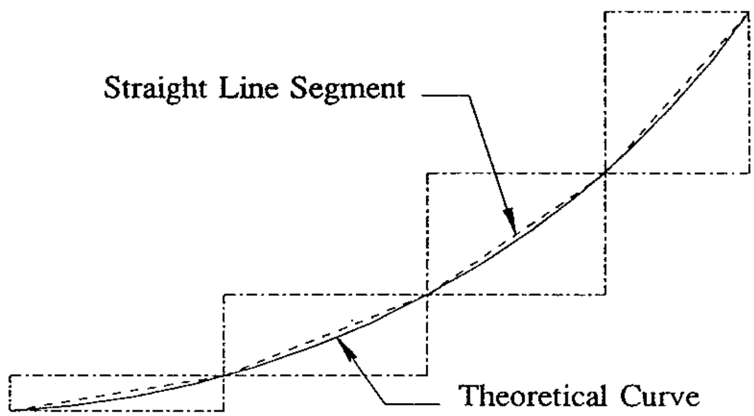

Figure 1 - Linear Interpolation Scheme

The current software algorithms begin this process with a functional description of the surface to be generated. This functional description is required to permits the calculation of first and second derivatives. From these, the instantaneous radius of curvature can be calculated for any point within the range of the aspherical surface. By evaluating the instantaneous radius of curvature over this range a minimum instantaneous radius can be found. Using this radius and a maximum interpolation error the spacing of points that form the linear segments is determined. Thus, the total number of points required can be approximated by dividing the curve length by this linear segment length. The effects of finite tool tip geometry is then calculated for each of these points and applied to create the tool path. When this fit is required to be improved a smaller linear interpolation error is used and a more dense selection of points results. The limit on the number of points used has been the machine control system's processor speed (update considerations) and data memory storage.

During recent work on the description of machine tool motions for the *LAST* test[2] it was found that reducing the linear interpolation error did not necessarily improve the accuracy of the tool path description. The argument that more and more points increases accuracy is based on a mythical machine tool control system that permits infinite position programmability. Real machine tool control systems restrict the definition of these points to a grid of minimum programmability. A machine tool position can only be specified to a finite number of significant digits. This quantization will adversely effect the tool path's accuracy when the linear interpolation error becomes smaller than this programmability level. Figure 2 shows how shorter segments provide a less accurate fit to a theoretical curve than a more widely spaced set of points in the limit of quantized programmability.

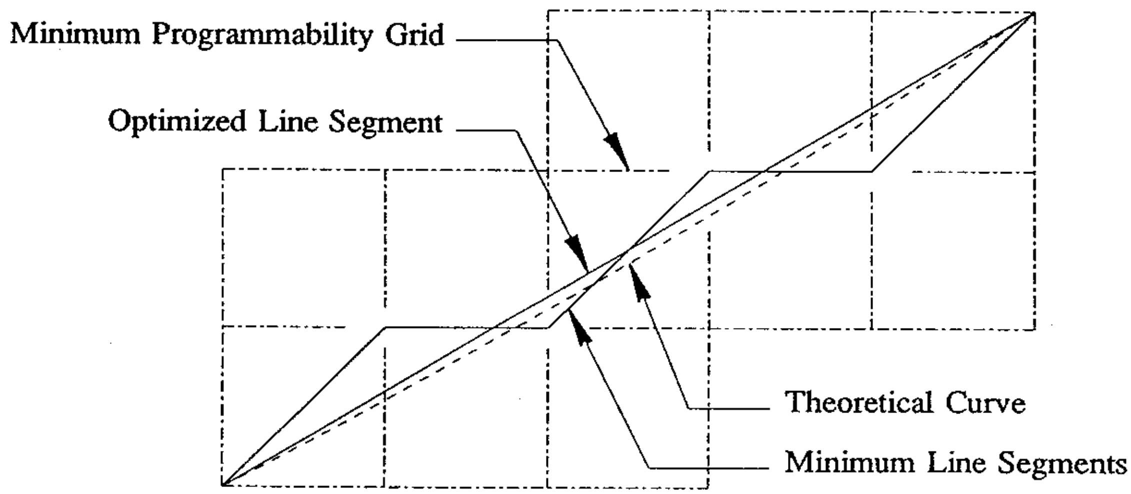

Figure 2 - Minimum Programmability Effects On Optimized Tool Path Descriptions

A new algorithm has be constructed that takes this quantized programmability into account. This algorithm searches along the theoretical curve in the vicinity of the usual end point (but always to shorten the segment) for the closest intersection between the curve and the grid of minimum programmability. When the closest point is found this point then becomes the end point for the current straight line segment and the starting point for the next segment. When the algorithm is applied after tool tip compensation the search is performed in an iterative fashion since the straightforward functional relationship becomes transcendental.

To demonstrate the benefits of this new algorithm consider the following example which for simplicity neglects tool tip compensation. Consider the generation of a paraboloid of revolution with a focal length of 125 mm and a diameter of 75 mm on a machine tool with 10 nm programmability and a maximum interpolation error of 1 nm. The functional form for this surface is given in equation 1. From this functional description derivatives can be determined and the functional form of the instantaneous radius of curvature expressed.

$$z = z(x) = \frac{x^2}{4f} \quad , \quad r(x) = \frac{\left(1 + \left[\frac{\partial z}{\partial x}\right]^2\right)^{\frac{3}{2}}}{\frac{\partial^2 z}{\partial x^2}} = 2f\left(1 + \frac{x^2}{4f^2}\right)^{\frac{3}{2}} \qquad (1)$$

By evaluating this function over the aspherical curve, the smallest instantaneous radius of curvature is found at the vertex of the curve and is equal to twice the focal length ($r_{min} = 2f$). Using this value for the minimum radius and the maximum interpolation error (ε) a minimum segment length is determined from equation 2 and calculated to be 0.0447 mm.

$$s \approx \sqrt{8 r_{min} \varepsilon} = \sqrt{16 f \varepsilon} \qquad (2)$$

Equation 3 describes the length along the aspheric curve. Solving this expression over the range of x values from the centerline to 37.5 mm (half the diameter) yields a distance along the curve of 37.64 mm. Dividing this value by the minimum segment length gives the approximate number of segments required to define the tool path, which is calculated to be 842.

$$L = \int_{x_{min}}^{x_{max}} \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + 1}\ \delta x = \int_{x_{min}}^{x_{max}} \sqrt{\frac{x^2}{4f^2} + 1}\ \delta x \qquad (3)$$

Using the point selection algorithm without the search feature the average distance between the theoretical curve and these 842 linear segments is 2.05 nm. This error is larger than the maximum interpolation error and is approximately half of the rounding error due to minimum programmability restrictions. Applying the new algorithm with a search length of one percent of the segment length results in a need to only increase the total number of linear segments in the path description by 4, but with an increase in the accuracy of the curve definition such that the average error is only 0.72 nm. This is a threefold improvement in the tool path description with a negligible increase in the number of points required!

The new search algorithm has proven to consistently improve the accuracy of tool path descriptions over a wide variety of aspherical surface forms. Because the algorithm creates the tool path description off-line and prior to surface generation it does not impact surface generation time. As the tolerances for surface accuracies improve this algorithm will provide a means to optimize the tool path descriptions for aspherical surface generation.

---

[1] Gerchman, Mark Craig, "Compensation of residual form errors in precision machined components", Proc. SPIE vol. 1573 (1991).

[2] Gerchman, Mark Craig and David H. Youden, "An evaluation of ultra-precise machine tool contouring performance: the Low Amplitude Sine Tracking (*LAST*) Test", Progress in Precision Engineering, Springer-Verlag (1991).