A Description Of Off-Axis Conic Surfaces
For Non-Axisymmetric Surface Generation

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Abstract

In this paper the equations that describe off-axis conic surfaces for a four axis approach to non-axisymmetric surface generation are derived. In addition, considerations that minimize the fourth axis range of motion are reviewed. An example, using a well known segmented astronomical mirror, is used to illustrate these considerations.

Introduction

A conventional single point diamond turning (SPDT) lathe uses two axes of motion to describe a curve in space. The curve is formed between the cutting edge of a diamond tool and the centerline of the work piece to be machined. It is the rotation of the work piece that generates, through the cutting action, a three dimensional surface of revolution. Because of the simplicity of these motions, the SPDT process has been in general restricted to the generation of axisymmetric surfaces.

The recent introduction[1] of four axis SPDT has provided a technique for the generation of non-axisymmetric surfaces. In addition to the orthogonal two axes of motion (*i.e.* x and z axes) available on a SPDT lathe, four axis machining utilizes two additional motions. The first additional motion is created by encoding the work holding spindle (*i.e.* φ axis) so that the machine tool controller has access to the angular position of the work piece during machining. The fourth axis of motion (*i.e.* z' axis) is an additional linear axis that holds the SPDT tool and has a limited range of motion, but, is capable of extremely rapid positioning. By coordinating the motion of the tool as a function of the work piece's angular position a non-axisymmetric surface can be generated.

One application of this technology is the machining of off-axis segments for large mosaic mirrors. The traditional technique for SPDT these surfaces has been to position the segment the appropriate distance from the work holding spindle axis and to machine the surface as a part of a full parent surface of revolution. This technique restricts how far off-axis a segment can be from the parent axis to the maximum machine tool swing geometry. This restriction is principally on the distance the segment is off-axis and not it's physical size, i.e. even small segments would require prohibitively large machine tools if they were very far off-axis. By using a four axis approach to the machining of these surfaces, the segments could be positioned on axis with the correct off-axis surfaces generated by four axis non-axisymmetric machining.

One element of this four axis approach to non-axisymmetric surface generation is to describe the surface geometry of these segments in an appropriate fashion. This paper describes the geometry of off-axis conic surfaces in a way that is geometrically exact and is capable of the rapid "real-time" solution required by four axis positioning at high rotational speeds. In addition, considerations that minimize the range of motion of the fourth axis are reviewed. The geometries of the off-axis segments of the Keck Telescope[1] primary mirror are used as examples to describe these considerations.

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Off-Axis Surface Geometry

Most aspheric off-axis surfaces are described by a sagitta equation in a coordinate system referenced to the parent surface; such as the following:

$$z=\frac{c\rho^2}{1+\sqrt{1-(k+1)c^2\rho^2}}+a_1\rho+a_2\rho^2+a_3\rho^3+a_4\rho^4+\cdots \quad (1)$$

This equation describes a conic surface of revolution modified by simple polynomial terms in a cylindrical coordinate system, with the origin of the coordinate system coincident with the vertex of the aspheric surface. The z axis is the sagitta axis and is collinear with the asphere's axis of rotation. For the non-axisymmetric generation of an off-axis section a description of the section geometry in a polar coordinate system centered on the off-axis surface is required. For the modified general aspheric surface this description requires the solution of transcendental equations. Fortunately, the case of the general conic of revolution can be solved in closed form and will be considered here. A closed form description has the advantage that in a four-axis control system the required motions can be coordinated in a "real time" environment.

Approximate representation of off-axis conic surfaces of revolution have been described in the literature[3] for surfaces fabricated by deformation techniques. Approximations for general non-axisymmetric surface fabrication based upon Zernike polynomials has also been described.[4] An exact description for the off-axis paraboloid with restrictions on orientation has been reported by Thompson.[5] The derivation presented here for the general off axis conic surface, tilted in the meridional plane at an arbitrary angle, follows the method of Thompson.

Thompson's method can be divided into four sections. First, the off axis geometry is described in the coordinate system of the parent conic surface, with the center of the off-axis segment along the x axis. The description is then translated to a second coordinate system, whose origin coincides with the center of the off-axis segment. The description is then rotated in the meridional plane to a third coordinate system. Finally, this description is converted from a Cartesian form to a cylindrical representation. Figure 1 shows the coordinate systems used.

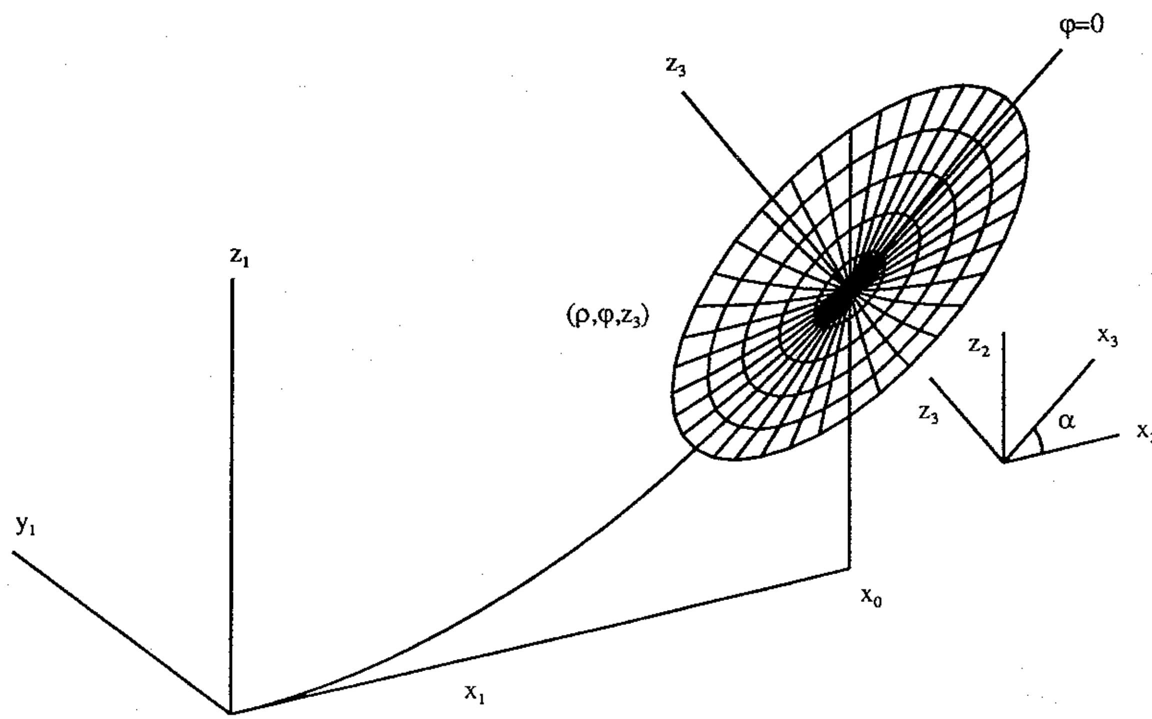


Figure 1 Off-Axis Geometry

The derivation begins with the description of the off-axis conic surface in the coordinate system of the parent surface of revolution. This initial description is given in equation 2, where: r is the paraxial radius of curvature of the parent conicoid and k is the conic constant.

$$x_1^2+y_1^2-2rz_1+(k+1)z_1^2=0 \tag{2}$$

By selecting a point $(x_0,0,z_0)$ on the off-axis surface a second coordinate system can be created, by using equation set 3, to translate the origin from the parent vertex to this new point. By direct substitution, a new description of the centered off-axis surface can be written. After simplification this yields equation 4.

$$\begin{aligned} x_1&=x_2+x_0 \\ y_1&=y_2 \\ z_1&=z_2+z_0 \end{aligned} \tag{3}$$

$$2x_0x_2+x_2^2+y_2^2-2rz_2+2(k+1)z_0z_2+(k+1)z_2^2=0 \tag{4}$$

A third coordinate system can be formed by rotating the off-axis centered system through a meridional angle, α, via equation set 5. After grouping like terms and simplifying, the description of the off-axis surface can be re-written in the form of a quadratic, equation 6, in the new sagitta direction z_3.

$$\begin{aligned} x_2&=x_3\cos(\alpha)-z_3\sin(\alpha) \\ y_2&=y_3 \\ z_2&=x_3\sin(\alpha)+z_3\cos(\alpha) \end{aligned} \tag{5}$$

$$\begin{aligned} &z_3^2[1+k\cos^2(\alpha)]+ \\ &z_3[2kx_3\cos(\alpha)\sin(\alpha)-2x_0\sin(\alpha)-2r\cos(\alpha)+2(k+1)z_0\cos(\alpha)]+ \\ &[2x_0x_3\cos(\alpha)+x_3^2+y_3^2-2rx_3\sin(\alpha)+2(k+1)x_3z_0\sin(\alpha)+kx_3^2\sin^2(\alpha)]=0 \end{aligned} \tag{6}$$

A final transformation of coordinate systems from Cartesian to cylindrical, using equation set 7, yields the quadratic sagitta description, equation 8, in the final orientation required.

$$\begin{aligned} x_3&=\rho_3\cos(\varphi) \\ y_3&=\rho_3\sin(\varphi) \\ z_3&=z_3 \end{aligned} \tag{7}$$

$$\begin{aligned} &z_3^2[1+k\cos^2(\alpha)]+ \\ &z_3[\rho\cos(\varphi)(2k\cos(\alpha)\sin(\alpha))-2x_0\sin(\alpha)-2r\cos(\alpha)+2(k+1)z_0\cos(\alpha)]+ \\ &[\rho\cos(\varphi)(2x_0\cos(\alpha)-2r\sin(\alpha)+2(k+1)z_0\sin(\alpha))+\rho^2+\rho^2\cos^2(\varphi)(k\sin^2(\alpha))]=0 \end{aligned} \tag{8}$$

After considerable simplification, the solution to this quadratic can be written in the form of equation 9. The choice of this form permits the dependence of z_3 on ρ and φ to be clearly examined. The new constants created are defined in equations 10 through 15.

$$z_3=d_1+d_2\rho\cos(\varphi)\pm\left[d_3+d_4\rho\cos(\varphi)+d_5\rho^2+d_6\rho^2\cos^2(\varphi)\right]^{1/2} \tag{9}$$

$$d_1 = \frac{x_0\sin(\alpha)+r\cos(\alpha)-(k+1)z_0\cos(\alpha)}{1+k\cos^2(\alpha)} \tag{10}$$

$$d_2 = \frac{-k\cos(\alpha)\sin(\alpha)}{1+k\cos^2(\alpha)} \tag{11}$$

$$d_3 = d_1^2 \tag{12}$$

$$d_4 = 2d_1d_2 - 2\frac{x_0\cos(\alpha)-r\sin(\alpha)+(k+1)z_0\sin(\alpha)}{1+k\cos^2(\alpha)} \tag{13}$$

$$d_5 = \frac{-1}{1+k\cos^2(\alpha)} \tag{14}$$

$$d_6 = d_2^2 - \frac{k\sin^2(\alpha)}{1+k\cos^2(\alpha)} \tag{15}$$

The restrictions on the use of this form are very few. Since there is a common term in each constant's denominator, *i.e.* $1+k\cos^2(\alpha)$, if its value is zero the form fails. This can happen for hyperboloid segments tilted at certain angles and for the unique case of a tilt free paraboloid. The choice of sign for the radical in equation 9 is determined by the sign of this denominator term. The form also fails when the quantity under the radical is negative. This happens when the value of ρ exceeds the range of the conicoid.

Factors That Minimize Fourth Axis Motion

This description of the off-axis conic surface is appropriate for those manufacturing schemes that use only three (*i.e.* x,z,φ) axes of motion.[6] It does not, however, provide the required separation of sagitta motions (i.e. z from z′) needed for a four axis machining scheme. Because of the restricted motion range of the fourth axis it is necessary to divide the total sagitta motion, z_3, in a way that minimizes the fourth axis movement. This can be accomplished by selecting an optimum baseline of motion for the slow z axis and using the motion of the fast z′ axis to produce the required non-axisymmetric motion. In addition to the selection of an optimum baseline, the selection of an optimum meridional tilt angle, α, can also minimize the fourth axis motion.

To illustrate how the choice of optimum tilt angle and baseline subtraction effect minimizing fourth axis motion the off-axis segments of the Keck Telescope primary mirror will be used as an example. Figure 2 shows the well known tessellation of the Keck primary mirror.[7] The thirty six off-axis segments of the mirror can be grouped into six distinct surface shapes. Note that mirror segments of type 4 and 5 represent right and left hand versions of the same geometry. When viewed along the axis of rotation of the hyperboloid primary, the outline of each projected section is hexagonal. The edge length of each hexagon is 0.90 meters. The primary mirror for this Ritchey-Chrétien telescope has a paraxial radius of curvature of 34.974 meters and a conic constant of -1.003683. The geometry of the sixth segment group will be used for this example because it is the most non-axisymmetric.

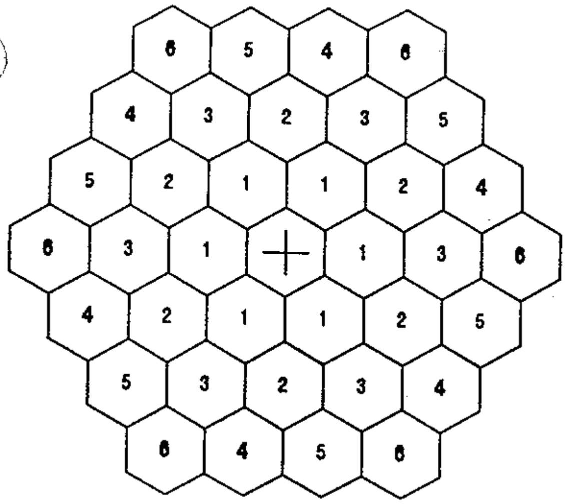
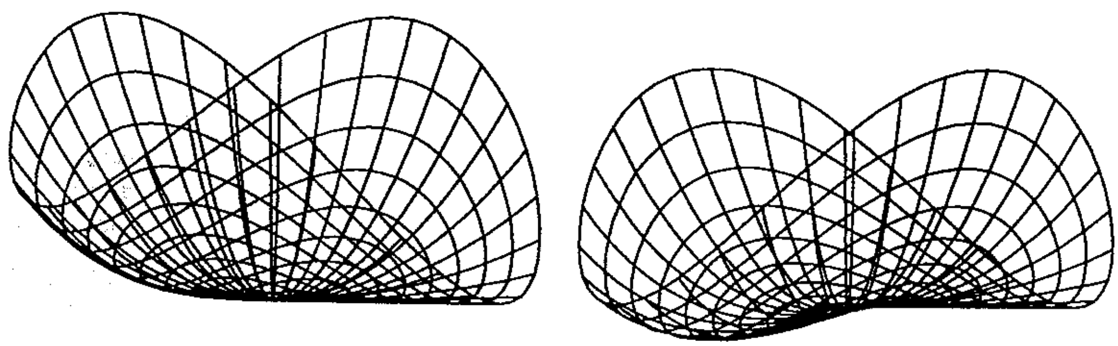


Figure 2 Keck Primary Mirror Tessellation

A first estimate for the optimized meridional tilt angle is the slope of surface at the off-axis center point $(x_0,0,z_0)$ as viewed in the coordinate system of the original parent surface. While this estimate tends to minimize the non-axisymmetric tool motion in the vicinity of the center point it fails to optimize the non-axisymmetric motion at any radial distance away from that point. By choosing a tilt angle that balances the figure in the meridional plane at the extreme radial positions, the overall non-axisymmetric geometry is minimized. The difference between the optimum slope and the original parent slope for the Keck segments are within a few seconds of arc. The effect on fourth axis motion is typically a reduction of twenty percent for the optimized tilt over the original slope. Figure 3 shows an anamorphically distorted view of the fourth axis motions for the outside segment of the Keck telescope segment with an optimized tilt angle and with the slope of the parent conic used. Note that the wire diagrams are for the minimum inscribed rotational swing required to generate the surfaces. The baseline used to generate these figures has the z axis set to the value of z_3 at $\varphi=0$ for each value of ρ. Table 1 contains the tilt correction from the parent slope used to optimize the six geometries of the Keck primary.



Slope of Parent Conic at x_0 Optimized Choice for α

Figure 3 Z′ Fourth Axis Motion - Outside Keck Segment ($\varphi=0°$ Baseline Subtracted)

row	center position x_0	tilt correction $\delta\alpha$	fourth axis motion z'
1	1558.8 mm	-3.10 arc sec	0.0235 mm
2	2700.0 mm	-5.31 arc sec	0.0701 mm
3	3117.7 mm	-6.10 arc sec	0.0932 mm
4	4124.3 mm	-7.93 arc sec	0.1617 mm
5	4124.3 mm	-7.93 arc sec	0.1617 mm
6	4676.5 mm	-8.88 arc sec	0.2067 mm

Table 1 Four Axis Parameters For The Keck Telescope Primary Mirror

The choice of an optimum baseline is straightforward. For each radial position the sagitta motion, z_3, has a maximum and a minimum value. By selecting the baseline to be the mean of these two values the range of the fourth axis motion remains centered about a single position. Figure 4 shows two views of the outside keck segment with both the optimum tilt angle and optimum baseline subtracted. The wire frame model on the left is the surface as viewed from the parent axis and on the right from the orthogonal perspective. Values for the optimized fourth axis motions for all six Keck geometries are shown in table 1. It should be noted that for all of these segments the range of motion of the fourth axis required is very small in comparison to the dimensions of the segments themselves.

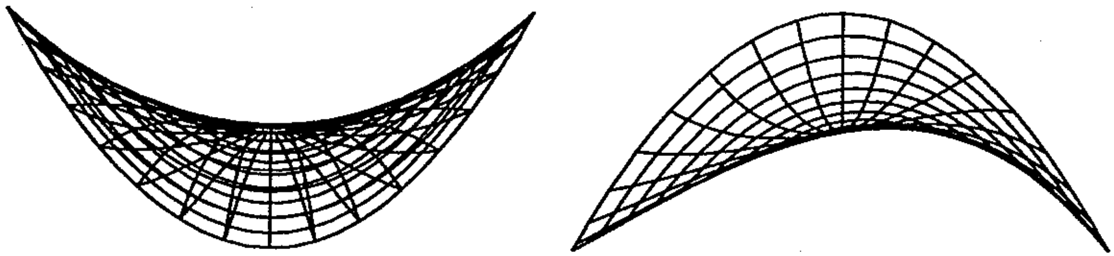


Figure 3 Z′ Fourth Axis Motion - Outside Keck Segment (Optimized Baseline Subtracted)

Conclusion

This paper has provided the derivation of the equations that describes the general off-axis conic surface. The equations derived are suitable for the four axis generation of these surfaces in a non-axisymmetric fashion. The factors that minimize the range of motion have been reviewed.

References

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